

Fluid Flow and Heat Transfers in Wellbores

Second Edition

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Dedicated to
Nilufar and Kumkum

Preface

Fluid flow in wellbores occurs during various phases of a well's life. Our ability to optimize each flow process depends largely on grasping the underlying physics so that we can mathematically describe the process involved. At a well's inception, drilling operations require mud circulation, causing considerable heat exchange between the colder fluid and the warmer formation. In the event of a blowout, because of lost circulation or unexpected overpressured zone, we encounter transient two-phase flow as the formation interacts with the wellbore prematurely. When we initiate flow from the formation by design, such as in a drillstem or production test, flow of either a single-phase or a multiphase fluid occurs. As the fluids ascend the wellbore, the warm formation fluid begins to exchange heat with the colder formation above it. Therefore, heat flow is always coupled with fluid flow in actual wellbores.

In this book, we attempt to address the coupled fluid/heat flow issue as encountered in many practical production/operation problems, including in drilling. Both steady- and unsteady-state transport problems are considered. Even when steady fluid flow is maintained during circulation, injection, or production modes, unsteady heat transport in the formation occurs nonetheless. Fluid circulation during drilling and workover operations and injection of annular gas in a gas lift operation are cases in point. We also examine fully transient processes of fluid and heat flows, such as those in drillstem or production testing.

Before we undertake a detailed analysis of each operational problem, we introduce the reader to some basic concepts, starting with the rudiments of single-phase flow (Chapter 1) and moving to more-complex issues of two-phase flow modeling (Chapters 2, 3, and 4). Chapter 5 is a new addition to encapsulate the lessons learned from application of the models in various field settings discussed in earlier chapters. Then, the principles of heat conduction in the formation, as well as the elements of fluid flow and the associated heat flow, are discussed in Chapter 6, followed by a new Chapter 7 showing various applications in coupled fluid/heat flow problems.

Chapter 8 introduces analytical models in various bundled-tube settings, where production and/or injection occurs in multitubing configurations. Chapter 9 presents aspects of modeling transient fluid and heat flow problems. Well integrity issues pertaining to annular pressure buildup, sustained casing pressure, and gas lift valves form the cornerstone of Chapter 10; both appropriate models and field examples exemplify the lessons learned from coupled fluid/heat flow modeling. Along with Chapters 5 and 7, Chapter 10 is new to this edition of the book.

Chapter 11 presents various analytical models pertaining to drilling operations. Here, simple analytical tools attempt to capture the evolution of wellbore temperature profiles in various operational settings, followed by an estimation of the static formation temperature, leading to determination of the geothermal gradient. Thereafter, estimating the spilled volume in uncontrolled wells is pursued for both gas and oil wells in a probabilistic framework.

Chapter 12 attempts to capture various aspects of production operations with surveillance data. In this context, production of organic solids, such as paraffins and asphaltenes, is addressed. Two field studies involving coupled modeling of reservoir/wellbore/surface network emphasize the lessons learned from a holistic approach. Finally, Chapter 13 discusses some aspects of production logging; flowmeter and temperature responses are the essence of this discussion.

Besides introducing three new chapters (Chapters 5, 7, and 10), this new edition of the book has updated all others wherever appropriate. Our expectation is that the three new chapters illuminate application of diverse flow problems that can be tackled with the basic principles learned. This edition is expected to serve a variety of readers, from advanced senior and graduate students to researchers and practicing engineers. The overall philosophy is to show not just how to solve a given problem but also why the recommended approach is preferred. Illustrative examples presented at the end of each major section reinforce the principles learned. In other words, striking a balance between theory and practice has led us to the avoidance of a proverbial cookbook approach.

Although over the last seven decades hundreds of papers have been written on the topics of multiphase flow and heat transfer, here we have attempted to present only those that pertain to solving some of the wellbore flow

problems. Therefore, this book does not treat either topic in great depth, but attempts to acquaint the reader with enough information so that practical oilfield flow problems can be tackled. In presenting various approaches to solving a problem, we have favored physical models, which have been verified with either laboratory or field data, or with both, over purely empirical correlations. However, in choosing mechanistic models, we have leaned toward a simpler approach, rather than delving into complex but rigorous solutions. Here, the motivation was to retain simplicity and engineering accuracy. In this context, we must point out that we have drawn heavily from our experience, both academic and applied, to present this material. In this compilation effort, our familiarity with our own material has taken precedence, notwithstanding objectivity. The citations mostly included accessible peer-reviewed papers, although hard-to-find classic papers required mention, given their seminal contributions.

We have not attempted to solve *all* the coupled fluid/heat flow problems in production operations. This second edition of the book is simply an attempt to capture a few more problems than those investigated earlier. Our ardent hope is that the presented material gives both the foundation and the tools for tackling a new class of problems.

Acknowledgments

In our quest to grasp various aspects of production operations as practiced in the oil field, we learned from others through personal interaction and through published work. Our colleagues, in both academia and the industry, have enriched our knowledge over the years. We are indebted to them all.

We would like to recognize a few who have made the real difference in our learning and the eventual compilation of the second edition of this book. Three noted practitioners reviewed three new chapters in this edition of the book. Specifically, we are indebted to Janvier Lissanon of BHP Billiton for reviewing Chapter 5, U. B. Sathuvalli of Blade Energy for scrutinizing Chapter 10, and Suri Suryanarayana, also of Blade Energy, for providing useful feedback on Chapter 7. We remain grateful to Cem Sarica of Tulsa University for reviewing the manuscript of the book's first edition. We express our sincere gratitude to our respective organizations, the University of North Dakota, Texas A&M University, and Chevron and Hess, for aiding our pursuit. SPE book editor Jane Eden's stewardship is much appreciated. We are immensely grateful to Harry Butler for doing a thorough painstaking job during the copy-editing phase of this project.

We are indebted to many graduate students who have made direct contributions to the book's cause over the years. Most notably, Xiaowei Wang and Dongqing Lin at the University of North Dakota contributed a great deal in developing many models presented in several chapters. We acknowledge the contributions of Bulent Izgec of Texas A&M University and Chevron to modeling thermal effects influencing pressure gauge response and annular pressure buildup. More recently, two Texas A&M University graduate students gave invaluable help: Tony Rocha-Valadez contributed to the modeling of sustained casing pressure and integrity of gas lift valves, and Ruochen Liu helped develop blowout models for gas and oil wells. A current Texas A&M University student, Gibran Hashmi, assisted in developing models for rate estimation from temperature or pressure data. Many field examples were drawn from hands-on experience that one of us (C. S. Kabir) has had in fields across many basins.

Several academics helped shape our understanding of this technology through their exemplary leadership. Graham Wallis of Dartmouth College formalized the drift-flux approach for two-phase flow modeling. The late Abraham Dukler of the University of Houston and the late Hank Ramey Jr. of Stanford University laid the foundation for mechanistic modeling of two-phase flow and wellbore heat transfer, respectively. We were inspired by the body of work that James Brill of the University of Tulsa and Khalid Aziz of Stanford University had contributed to the field.

Last, our family members, especially our spouses, deserve particular mention for their encouragement and fortitude. Their extraordinary understanding allowed us to steal countless hours from family time so that we could complete this edition of the book. Finally, our parents taught us values that inspired us to compile this material. To this end, we hope the reader finds this text stimulating and useful.

Rashid Hasan
Shah Kabir

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Chapter 1

Overview

1.1 Single-Phase Flow

Fluid flow, in a variety of forms and complexities, is a basic entity that must be dealt with in the production of hydrocarbons. In its rudiments, single-phase gas or oil production and water injection form the core of all flow problems. Therefore, Chapter 1 discusses the mechanical energy balance equation, which relates pressure drop to its various components for single-phase flow. Next, the components of total pressure drop—static, kinetic, and frictional—are discussed. In addition, flows in tubing/casing annuli and horizontal wells, which are of particular interest to petroleum engineers, are briefly discussed.

1.1.1 Mechanical Energy Balance. A simple one-dimensional (1D) analysis of single-phase gas or liquid flow is best made with the aid of a schematic, as shown in **Fig. 1.1**. The channel, inclined at an arbitrary angle α with the horizontal, shows upward flow of the fluid. We use the industry convention that the vertical axis z is positive in the downward direction. For the present, we consider only the steady-state case and assume that pressure, at any point in the cross-sectional plane normal to flow, remains the same. With these simplifications, we derive the momentum balance equation.

Conservation of Momentum. The sum of forces acting on the fluid element, shown in Fig. 1.1, equals the change of momentum of the fluid. The forces acting on the fluid element are those owing to pressure p , friction F , and gravity. Referring to the differential length dz of Fig. 1.1, we write

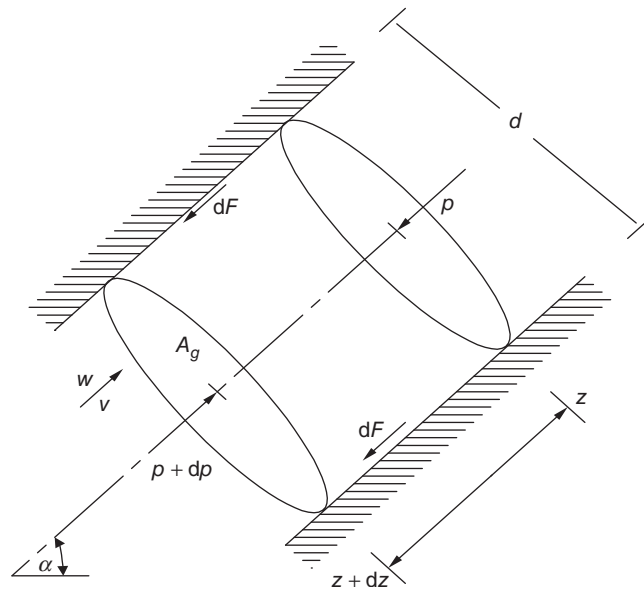


Fig. 1.1—Momentum balance for a fluid element.

$$pA - (p + dp)A + dF + A (dz) g \rho \sin \alpha = \text{change of fluid momentum.} \dots\dots\dots (1.1a)$$

If the fluid mass flow rate is w and its velocity is v , then its momentum equals wv . For the general case of transient flow, when both flow rate and velocity change along the flow direction, fluid momentum change is given by $(w + dw) (v + dv) - wv$. Therefore,

$$pA - (p + dp) A + dF + A (dz) g \rho \sin \alpha = (w + dw) (v + dv) - wv. \dots\dots\dots (1.1b)$$

Simplifying, we obtain

$$- Adp + dF + A (dz) g \rho \sin \alpha = wdv + vd w. \dots\dots\dots (1.2)$$

Usually, the mass flow rate is invariant; that is, $dw = 0$, leading to

$$- Adp + dF + A (dz) g \rho \sin \alpha = wdv. \dots\dots\dots (1.3)$$

Dividing both sides of Eq. 1.3 by Adz , we obtain

$$- (dp/dz) + (dp/dz)_F + g\rho \sin \alpha - (w/A) dv/dz = 0 \dots\dots\dots (1.4)$$

or

$$(dp/dz) = (dp/dz)_F + (dp/dz)_H + (dp/dz)_A, \dots\dots\dots (1.5)$$

where

$$- (dp/dz)_H = -g \rho \sin \alpha \dots\dots\dots (1.6)$$

and

$$- (dp/dz)_A = (w/A) d(v)/dz = \rho v d(v)/dz. \dots\dots\dots (1.7)$$

1.1.2 Components of Pressure Gradient. Eq. 1.5 shows the total pressure gradient is the sum of the frictional gradient $(dp/dz)_F$, the hydrostatic gradient $(dp/dz)_H$, and the accelerational gradient $(dp/dz)_A$. Of these three terms, perhaps the static gradient is the easiest to estimate because it only requires knowledge of the fluid density and well-deviation angle. Because gas density depends on pressure, the static term will vary along the well for gas wells. Usually such variation is small, and relatively simple equations of state can be used to account for it. To some extent, even for single-phase oil production, oil-density variation with well depth, owing to temperature and dissolved gases, must be taken into account. The same comments apply to the estimation procedure for the kinetic head (Eq. 1.7).

For incompressible flow (gases at very high pressures and liquids) in a straight pipe with no change in cross-sectional area, the change in fluid velocity with axial distance (dv/dz) is generally negligible. However, for gases at moderate and low pressures, and especially at high velocities, the kinetic energy loss can be a significant portion of the total pressure loss and must be accounted for properly. Computational complications that arise for gas flow have led to a number of correlations for calculating pressure drop in a wellbore. We recommend the widely used Cullender and Smith (1956) method for computing pressure drop in a gas well, when the accelerational component becomes negligible.

The frictional pressure gradient is generally represented by

$$(dp/dz)_F = f v^2 \rho / (2 g_c d). \dots\dots\dots (1.8)$$

In this book, we are using the Moody friction factor because of its popularity in the oil industry. The alternative is to use the Fanning friction factor, which is one-quarter the value of the Moody friction factor. In that case, Eq. 1.8 will change to $(dp/dz)_F = 2 f v^2 \rho / (g_c d)$, reflecting the lower value of the Fanning friction factor. Friction factor f depends on fluid turbulence and pipe roughness. The friction factor is usually expressed as a function of Reynolds number

$$Re = dv\rho/\mu \dots\dots\dots (1.9)$$

and roughness factor ϵ/d . The chart for friction factor as a function of Reynolds number with pipe roughness as a parameter is shown in **Fig. 1.2**, whereas **Fig. 1.3** presents the chart for estimating relative roughness. Note, ϵ/d represents the relative roughness in both figures.

At low Reynolds numbers ($Re < 2,100$), the flowing-fluid elements do not interact with each other, and the flow is called laminar. For laminar flow in either rough or smooth pipes, friction factor is inversely related to Reynolds number,

$$f = \frac{64}{Re} = \frac{64\mu}{d\nu\rho}, \dots\dots\dots (1.10a)$$

when

$$Re < 2,100. \dots\dots\dots (1.10b)$$

At high Reynolds numbers ($Re > 4,000$), the flow is termed turbulent. During turbulent flow, the friction factor depends on both Reynolds number and pipe roughness. For smooth pipes, such as plastic pipes and tubulars coated with polyvinyl chloride lining, friction factor can be estimated reliably from the Blasius equation,

$$f = 0.32 (Re)^{-0.25}, \dots\dots\dots (1.11)$$

when

$$Re > 2,100. \dots\dots\dots (1.12)$$

For very high Reynolds numbers ($Re > 50,000$), Eq. 1.11 is slightly modified as $f = 0.184 (Re)^{-0.2}$.

Eq. 1.11, of course, is invalid for rough pipes, which need the use of a chart, such as that in Fig. 1.2. Although a chart is useful for all types of pipe roughness, chart reading is tedious and is not easily amenable to computer calculations. A number of equations, relating friction factor to Reynolds number and pipe roughness, have been proposed over the years and are in fair agreement with the original friction factor charts. We recommend the following expression proposed by Chen (1979), modified to yield Moody friction factor

$$f = \frac{4}{\left[4 \log \left(\frac{\epsilon/d}{3.7065} - \frac{5.0452}{Re} \log \Lambda \right) \right]^2}, \dots\dots\dots (1.13)$$

where ϵ is pipe roughness and the dimensionless parameter Λ is given by

$$\Lambda = \frac{(\epsilon/d)^{1.1098}}{2.8257} + \left(\frac{7.149}{Re} \right)^{0.8981} \dots\dots\dots (1.14)$$

Unlike many other expressions, which require iterative solutions for the friction factor, Eq. 1.13 is explicit and, therefore, computationally efficient. de Nevers (2004, p.187) suggested a simpler explicit formulation, with somewhat lesser accuracy, given by

$$f = 0.0055 \left\{ 1 + \left[20,000 \left(\frac{\epsilon}{d} \right) + \frac{10^6}{Re} \right]^{1/3} \right\} \dots\dots\dots (1.15)$$

More recently, Ghanbari et al. (2011) have presented a similar explicit formulation with improved accuracy, given by

$$f = 4 \left\{ -1.52 \log \left[\left(\frac{\epsilon/D}{7.21} \right)^{1.042} \right] + \left(\frac{2.731}{Re} \right)^{0.9152} \right\}^{-2.169} \dots\dots\dots (1.16)$$

Note that Eqs. 1.15 and 1.16 have been modified to represent Moody friction factor. The evaluation of various terms in Eq. 1.13 is relatively easier for flow of single-phase fluids, even for gases, than for two-phase mixtures. In the latter case, estimating the average density and friction factor can be challenging because these are complex

functions of fluid properties and flow conditions. Chapter 2 discusses various approaches taken to evaluate these entities in two-phase flow.

1.2 Flow in Nonisothermal Systems

Fluid temperature in the wellbore often varies significantly with depth, and sometimes with time. Many of the fluid properties that influence pressure drop, such as density and viscosity, are greatly influenced by the fluid temperature. Therefore, we cannot overemphasize the importance of accurate fluid temperature estimation as a function of well depth and production or injection time. This calculation can be performed by a proper energy balance on the fluid/wellbore system, as shown in Chapter 6. For single-phase liquid flow, the expression for fluid temperature, T_f , may be simplified with minimal inaccuracy to

$$T_f = T_{ei} + (1 - e^{-L_R}) g_G \sin \alpha / L_R \dots \dots \dots (1.17)$$

where the parameter L_R , which is a function of wellbore heat-transfer coefficient U and formation heat conductivity k_e , is defined by

$$L_R = \frac{2\pi}{c_p W} \left[\frac{r U k_e}{k_e + (r U T_D)} \right] \dots \dots \dots (1.18)$$

In Eq. 1.18, T_D represents dimensionless temperature, which is a function of dimensionless producing time, $t_D = \alpha t / r_{wb}^2$.

$$T_D = \ln \left[e^{-0.3t_D} + (1.5 - 0.3719e^{-t_D}) \sqrt{t_D} \right] \dots \dots \dots (1.19)$$

For a complete discussion of Eqs. 1.17 through 1.19, please refer to Chapter 6.

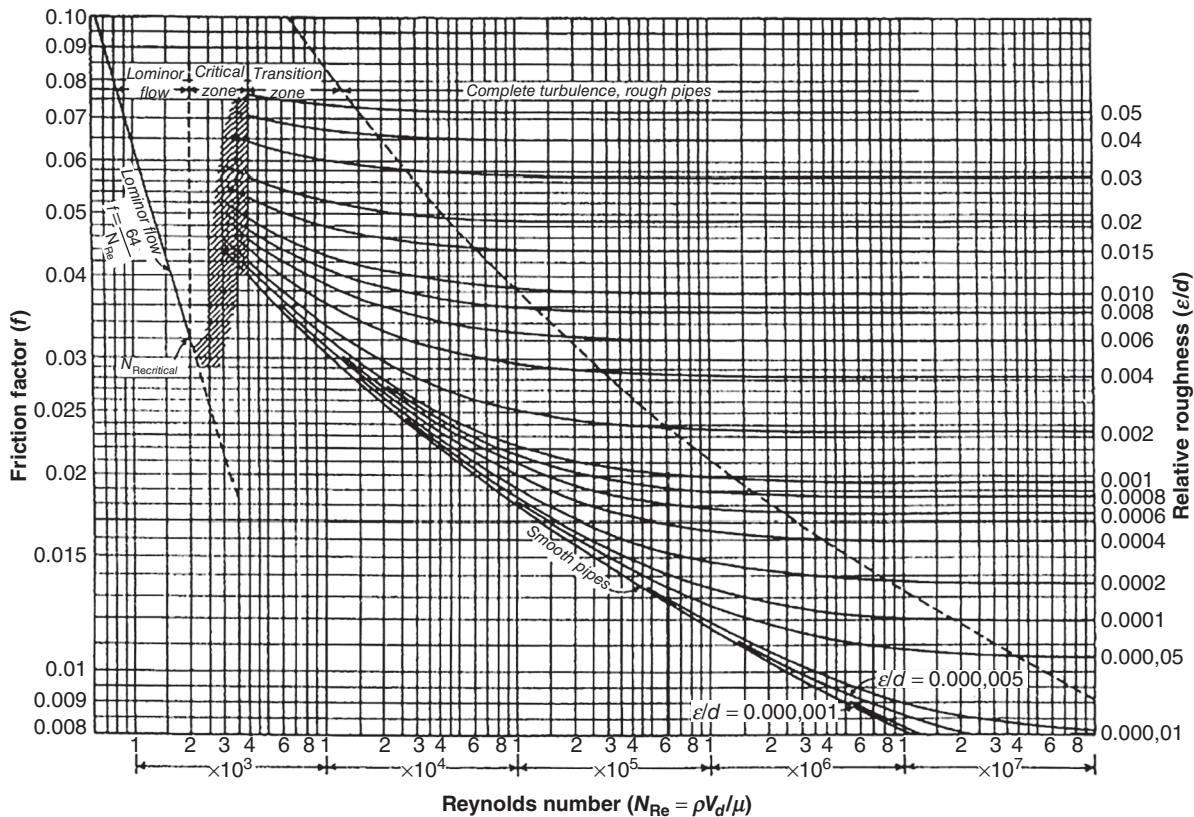


Fig. 1.2—Moody friction factor chart for turbulent flow (Moody 1944).

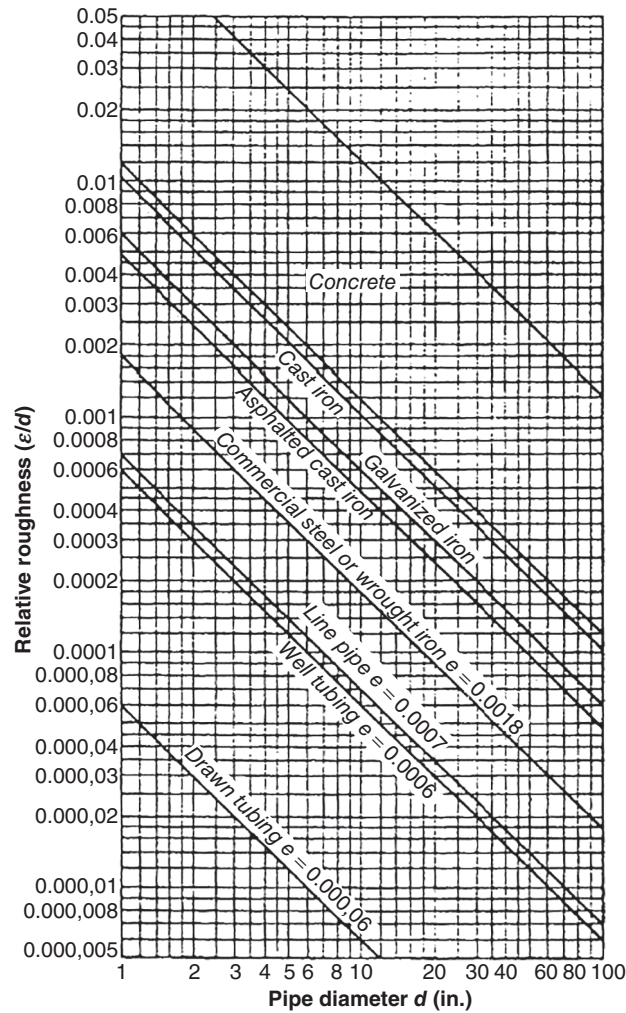


Fig. 1.3—Relative roughness of pipes (Moody 1944).

1.3 Flow in Annulus

Although flow through a tubing string is the most common configuration, drilling, as well as many completions, dictates modeling for flow up the tubing/casing annulus. The presence of two walls makes flow through an annulus different from that through ordinary circular strings. The classical work of Bird et al. (2002) shows that Eq. 1.8 is also applicable for such geometry, although the correlation for friction factor must be modified to reflect greater wall shear. For laminar flow in a concentric annulus, the Moody friction factor f_{CA} is given by Bird et al. (2002) as

$$f_{CA} = \frac{64}{Re} \frac{(1-K)^2}{\left[\frac{1-K^4}{1-K^2} - \frac{1-K^2}{\ln(1/K)} \right]}, \dots \dots \dots (1.20)$$

where K is the diameter ratio, d_i/d_o . Following the studies of Gunn and Darling (1963) and Caetano et al. (1992a), we recommend expressing turbulent flow in a concentric annulus as

$$\frac{1}{\left\{ f_{CA} \left(\frac{F_p}{F_{CA}} \right)^{0.45 \exp[-(Re-3000)/10^6]} \right\}^{0.5}} = 4 \log \left(\text{Re} \left\{ f_{CA} \left(\frac{F_p}{F_{CA}} \right)^{0.45 \exp[-(Re-3000)/10^6]} \right\}^{0.5} \right) - 0.4, \dots \dots \dots (1.21)$$

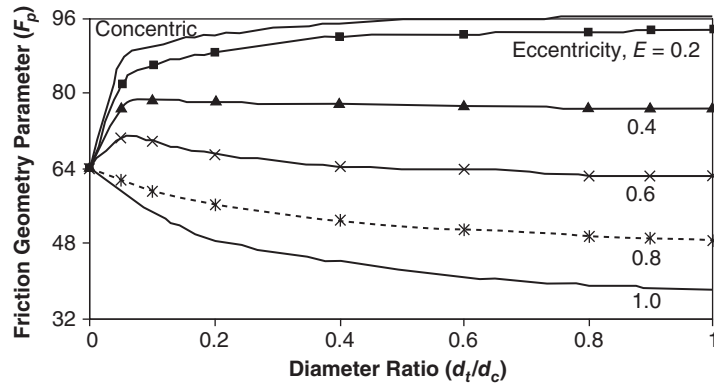


Fig. 1.4—Friction geometry parameter for concentric and eccentric annuli.

where F_p is the laminar-flow friction factor geometry parameter and F_{CA} is the ratio of friction factor for the annulus to that of a circular channel with the same d_c . Therefore, from Eq. 1.21, F_p for a concentric annulus is given by

$$F_p = \frac{(1 - K)^2}{\left[\frac{1 - K^4}{1 - K^2} - \frac{1 - K^2}{\ln(1/K)} \right]} \dots \dots \dots (1.22)$$

Understanding mud flow behavior in horizontal and near-horizontal drilling requires fluid flow through eccentric annuli. For such flows, eccentricity (E) is defined as

$$E = D / (d_c - d_t), \dots \dots \dots (1.23)$$

where D is the distance between the pipe centers. The values of F_p , as a function of K and E , are shown in **Fig. 1.4**. For an eccentric annulus, the friction factor equation is similar to Eq. 1.20:

$$f_{ECA} = \frac{4}{\text{Re}} \frac{4(1 - K)^2}{\xi \sinh^4 \eta_o} \dots \dots \dots (1.24)$$

In Eq. 1.24, η_o incorporates the effect of eccentricity factor E , and ξ is a function of η_o . A complete treatment of flow through eccentric annuli is beyond the scope of this book; for further details, the reader is directed to Caetano et al. (1992a). Chapter 4 discusses two-phase flow in an annular geometry.

1.4 Flow in Horizontal Wells

The interest in horizontal wells stems from significant increases in productivity and ultimate recovery in certain cases, such as unconventional reservoirs. Initial efforts by Dikken (1990) and Novy (1995) to couple the wellbore with the reservoir using analytic approaches considered frictional effects only. In other words, fluid ingress along the well length leading to momentum and related effects was ignored in those formulations.

Estimating pressure drop in horizontal wells presents a number of difficulties. First, pipe-surface roughness is a difficult entity to discern because of perforations along the well length in a cased borehole. Because most completions occur openhole, complexity increases significantly to ascribe a friction factor for an ill-defined surface—that is, the formation. The second factor revolves around fluid influx or changes in momentum that occur along the well length.

Experimental studies of Asheim et al. (1992), Ouyang et al. (1998), and Yuan et al. (1999) in perforated horizontal pipes, allowing fluid ingress along the well length, led to the development of several friction factor correlations. Of these, the results of Ouyang et al. (1998) and Yuan et al. (1999) are noteworthy.

Ouyang et al. (1998) presented the following Moody friction factor correlations for laminar and turbulent flows, respectively.

$$f = \frac{64}{\text{Re}} \left(1 + 0.04304 \text{Re}_w^{0.6142} \right) \dots \dots \dots (1.25)$$

and

$$f = f_o \left(1 - 0.0153 \text{Re}_w^{0.3978}\right), \dots \dots \dots (1.26)$$

where f_o is the no-wall-flow friction factor, which can be estimated from Eq. 1.13. Note, Re_w represents the wall Reynolds number, which is based on the pipe inner diameter (ID) and equivalent inflow velocity per unit wellbore length.

A somewhat different approach led Yuan et al. (1999) to obtain the following expression for the total or apparent friction factor, f_T (Moody friction factor), for fluid ingress along the borehole.

$$f_T = a \text{Re}^{-b} + C_n 2d\phi \frac{q_i}{q_a}, \dots \dots \dots (1.27)$$

where

$$a = 10219.5\phi - 3.25 \frac{q_i}{q_a} - 8.87 \times 10^{-4} \phi^2 + 5.37 \times 10^{-2} \phi - 0.075. \dots \dots \dots (1.28)$$

and

$$b = \left(-1.24 \times 10^5 \phi^{-3.075} + 42.4\right) \left(\frac{q_i}{q_a}\right)^2 + 1.577 \times 10^3 \phi^{-2.63} \frac{q_i}{q_a} - 5 \times 10^{-4} \phi^2 + 2.31 \times 10^{-2} \phi + 0.085. \dots \dots \dots (1.29)$$

For $(q_i/q_a) < 0.02$, $C_n = 2.3$, and for $(q_i/q_a) > 0.02$, C_n is given by

$$C_n = 4.25 \left(\frac{q_i}{q_a}\right)^{-0.099} \dots \dots \dots (1.30)$$

Experience shows that pressure drop in horizontal wells becomes important in high-transmissivity reservoirs, where the pressure drop in the wellbore becomes comparable to that in the formation. When the wellbore pressure drop becomes important, in most cases, the frictional component becomes the dominant mechanism. Chapter 4 discusses two-phase flow in horizontal wells.

Summary

The objective of this introductory chapter is to acquaint the reader with the rudiments of single-phase flow, which forms the backbone for understanding the mechanics of two-phase flow. Here, we attempted to capture some elements of fluid flow through conduits of various complexities, such as annuli and horizontal wells, and when fluid flow is accompanied by heat flow. Subsequent chapters discuss these elements in detail.

Nomenclature

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| a = parameter defined by Eq. 1.28, dimensionless | d_c, d_t = casing or tubing diameter, in. |
| A = cross-sectional area for fluid flow, ft ² | D = distance between pipe centers in Eq. 1.23, ft |
| A_g, A_l = cross-sectional area available for gas or liquid to flow, ft ² | E = eccentricity factor, dimensionless |
| b = parameter defined by Eq. 1.29, dimensionless | f = friction factor, dimensionless |
| c_p = heat capacity, Btu/(lbm-°F) | f_o = no-wall friction factor, dimensionless |
| C_n = parameter defined by Eq. 1.27, dimensionless | f_{CA} = friction factor of concentric annulus, dimensionless |
| d = pipe or well diameter, in. | f_{ECA} = friction factor of eccentric annulus, dimensionless |
| | f_T = apparent friction factor, dimensionless |

F = force, lbf	t_D = dimensionless time, $\hat{\alpha}t/r_w^2$ ($= 2.64 \times 10^{-4}kt/\phi\mu c_r r_w^2$)
F_p = friction geometry parameter, dimensionless	T_{ei}, T_e = formation temperature at initial condi- tion or at any radial distance, °F
g = acceleration because of gravity, ft/sec ²	T_f = fluid temperature, °F
g_c = conversion factor, 32.17 lbf-ft/lbf-sec ²	T_D = dimensionless temperature $= (2\pi k_e)(T_{wb} - T_{ei})/Q$
g_G = geothermal gradient, °F/ft	T_{wb} = wellbore fluid temperature, °F
H = fluid enthalpy, Btu/lbm	U = overall-heat-transfer coefficient, Btu/(hr-°F-ft)
k = formation permeability, md	v = fluid velocity, ft/hr
k_e = earth conductivity, Btu/(hr-ft-°F)	w = mass flow rate of fluid, lbm/hr
K = annulus/tubing diameter ratio, dimensionless	z = any vertical well depth, ft
L_R = relaxation distance parameter, ft ⁻¹	Z = gas-law deviation factor, dimensionless
p = pressure, psi	α = wellbore inclination with horizontal, degrees
(dp/dz) = pressure gradient, psi/ft	Λ = parameter given by Eq. 1.14, dimensionless
$(dp/dz)_A$ = accelerational (kinetic) pressure gradi- ent, psi/ft	μ = oil viscosity, cp
$(dp/dz)_F$ = frictional pressure gradient, psi/ft	ε = pipe roughness factor, ft
$(dp/dz)_H$ = static pressure gradient, psi/ft	ϕ = perforation density, 1/ft
q_a = average flow rate over incremental length, ft ³ /hr	ρ = density, lbm/ft ³
q_i = influx rate from each perforation, ft ³ /hr	λ = parameter for Baker flow pattern map, dimensionless
r_{wb} = wellbore radius, ft	η = parameter used in Eq. 1.24
r_{to} = outside tubing radius, ft	
Re = Reynolds number ($= dv\rho/\mu$), dimensionless	
Re_g, Re_L = Reynolds number for gas ($= \rho_g v_g d / \mu_g$) or liquid phase ($= \rho_L v_L d / \mu_L$), dimensionless	
Re_m = Reynolds number for mixture ($= \rho_m v_m d / \mu_m$), dimensionless	
Re_w = wall Reynolds number, dimensionless	
t = production, injection, or circulation (mud) time, hours	

Subscripts

c = casing
o = oil
t = tubing
to = tubing outside
wb = wellbore