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Introduction

The objective of this textbook is to introduce the reader to the fundamentals of applied well test interpretation. The text focuses on the most basic well testing scenario: a single-well test on a well producing a single-phase fluid, from a single-layer, homogeneous reservoir. Although simple, this scenario illustrates most of the elements required for interpretation in more complex scenarios.

Chapter 1—Introduction to Applied Well Test Analysis opens with an overview of different types of well tests, common applications and objectives in well testing, and alternatives to conventional testing. The chapter continues with a review of reservoir rock and fluid properties and ends with a brief discussion of the effects of graphical scales on data presentation.

Chapter 2—Fluid Flow in Porous Media covers the assumptions on which the diffusivity equation is based, then introduces the concepts of superposition in space, superposition in time, and radius of investigation. The remainder of the chapter focuses on the applied topics of wellbore damage and stimulation, pseudosteady-state flow, and wellbore storage.

Chapter 3—Radial Flow Semilog Analysis introduces semilog methods for estimating permeability and skin factor from data in infinite-acting radial flow for both drawdown and buildup tests. The chapter also discusses classical methods of estimating average reservoir pressure using the semilog plot.

Chapter 4—Log-Log Type Curve Analysis discusses the Gringarten-Bourdet pressure and pressure derivative type curves and the log-log field data plot. The chapter discusses use of the log-log plot to qualitatively evaluate whether or not a well is damaged or stimulated, and to identify wellbore storage and the infinite-acting radial flow period. The chapter covers estimation of permeability, skin factor, and wellbore storage coefficient from the log-log field data plot without using type curves. The chapter closes with a discussion of some common methods used to calculate the logarithmic derivative from field data.

Chapter 5—Pressure Transient Testing for Gas Wells introduces the real-gas pseudopressure and pseudotime transforms and their normalized counterparts, adjusted pressure and adjusted time, to allow the use of methods developed for slightly compressible liquids to be used for analysis of gas well test data.

Chapter 6—Flow Regimes and the Diagnostic Plot introduces the common flow regimes and the use of the standard log-log and flow-regime specific diagnostic plots for flow-regime identification. For each flow regime, examples of one or more reservoir models that exhibit the flow regime are given.

Chapter 7—Bounded Reservoir Behavior covers the most common models of single-layer reservoir behavior. For each model, the flow regimes that may be exhibited and the order in which they occur are discussed.

Chapter 8—Variable Flow Rate History discusses various methods for treating a variable flow-rate history, from ignoring prior history for short buildups following long drawdowns to deconvolution. The chapter discusses the effects of some common types of boundaries on the shape of the log-log buildup test for different ways of plotting the data. The chapter then provides a spatial interpretation of a variable rate history as a pressure profile in the reservoir. The effects of rate history on a subsequent buildup are discussed, as are the differences between the drawdown and buildup responses for a well in a closed reservoir, a reservoir with a constant-pressure boundary, and a radial composite reservoir. A method for graphing the rate history preceding a buildup along with the pressure response during the buildup is introduced to help distinguish rate-history-induced features in the derivative from those caused by boundaries.

Chapter 9—Wellbore Phenomena addresses an issue that impacts any well test to one degree or another, yet has received only sporadic attention in the well testing literature. A number of different wellbore phenomena that

may affect the shape of the pressure response, such as changing wellbore storage, a rising or falling fluid interface, and completion cleanup, are discussed. In addition, other phenomena that affect the pressure response but have no impact on well productivity (such as pressure fluctuations from earth tides or daily changes in wellhead temperature, gauge problems, or data processing artifacts) are also addressed.

Chapter 10—Near-Wellbore Phenomena covers phenomena present in the near-wellbore area that do impact the well performance, including geometric skin factor for a perforated completion, a limited-entry or partial penetration completion, or a deviated well, and non-Darcy skin factor for both drawdown and buildup.

Chapter 11—Well Test Interpretation Workflow presents a recommended workflow (more accurately, a workflow framework or checklist) for well test interpretation. The major steps are the same for virtually any well test interpretation: collect the data, QC the data, identify flow regimes, select a reservoir model, estimate model parameters, and validate the results.

Chapter 12—Well Test Design Workflow presents a recommended workflow for well test design. As with well test interpretation, the major steps in well test design are the same for most situations: define the test objectives, collect data, estimate unknown reservoir properties, estimate test duration, estimate test flow rate, and determine flow rate sequence

Disclaimer. The phrases “recommended procedure,” “recommended practice,” or other similar phrases refer to procedures or practices recommended by the authors and do not imply endorsement by the Society of Petroleum Engineers.

Chapter 1

Introduction to Applied Well Test Interpretation

“‘Begin at the beginning,’ the King said gravely, ‘and go on till you come to the end: then stop.’”

—Lewis Carroll, *Alice’s Adventures in Wonderland* (1865)

“The last thing one settles in writing a book is what one should put in first.”

—Blaise Pascal, *Pensées*

1.1 Introduction

Well test interpretation is the process of obtaining information about a reservoir through examining and analyzing the pressure-transient response caused by a change in production rate. This information is used to make reservoir-management decisions. It is important to note that the information obtained from well test interpretation may be qualitative as well as quantitative. Identification of the presence and nature of a no-flow boundary or a down-dip aquifer is just as important as, if not more important than, estimating the distance to the boundary.

This textbook focuses on well test *interpretation* rather than well test *analysis*. Much of the literature on well test analysis focuses on finding solutions to the transient flow equation for new boundary conditions, using advanced tools such as Bessel functions, Laplace and Fourier transforms, and Green functions. Even applied well test analysis is often thought of as simply estimating reservoir properties by drawing straight lines or matching curves. Interpretation, in contrast, seeks first to understand the nature of the reservoir that produced the pressure response, and only then to quantify the physical properties that describe the reservoir.

Section 1.2 provides an overview of the types of well tests and applications and objectives of well test interpretation.

Section 1.3 gives a brief description of several alternatives to conventional well testing, including wireline formation testing, permanent monitoring, rate-transient analysis, log analysis, and core analysis.

Section 1.4 discusses well test interpretation as an inverse problem, where the system input (flow rate) and output (pressure response) are known, and the system (reservoir model) must be determined.

Section 1.5 introduces the three categories of well test interpretation methods, straight-line analysis, type-curve matching, and simulation/history matching.

Section 1.6 defines and discusses the major variables used in well test interpretation, including rock properties such as porosity, saturation, pore-volume compressibility, permeability, and net-pay or net sand thickness, fluid properties such as formation volume factor, viscosity, and compressibility, and other properties such as wellbore radius and total compressibility.

Section 1.7 discusses some of the properties of the more common graph scales used in well test interpretation.

After completing this chapter, you should be able to

1. List the major types of well tests and give the principal goal for each type.
2. List five alternatives to conventional well testing, and describe the advantages, disadvantages, and applications of each.
3. List the two major tasks of well test interpretation.
4. List the three classes of analysis methods used in well test interpretation.
5. Define and give typical values for each of the primary variables used in well test interpretation.
6. Describe the effect of graphing data on Cartesian, semilog, and log-log scales.

1.2 Applications of Well Testing

1.2.1 Test Types. There are a wide range of types of well tests, as well as a number of different ways of categorizing well tests. Tests may be classified by the way in which they are carried out, whether they involve one or multiple wells, whether the test is run by opening up a well for production or shutting in a producing well, whether the well is used for production or injection, and whether the well is an exploration well or a well in a mature field.

Deliverability Tests vs. Transient Tests. The primary objectives of deliverability tests are to obtain fluid samples, to determine well deliverability, to determine the well potential, or to develop inflow performance curves for system analysis. Primary objectives of transient tests may include to estimate in-situ permeability to oil or gas, to estimate average drainage area pressure, to evaluate stimulation treatment effectiveness, and to estimate drainage area and/or fluids in place.

Multiwell Tests vs. Single-Well Tests. In a multiwell test, the rate is changed at one well (the active well), and the pressure response is measured at one or more offset, or *observation*, wells. Multiwell tests are often more complex than single-well tests. Multiwell tests are usually run to quantify the degree of communication between wells or to estimate directional permeability.

In a single-well test, the rate is changed, and the pressure response is measured at the well being tested. Objectives of single-well tests may include to quantify the degree of damage or stimulation, to estimate the in-situ permeability, to estimate the drainage area or fluids in place, and to estimate the average drainage area pressure.

Buildup Tests vs. Drawdown Tests. In a drawdown test, the reservoir is initially at uniform pressure, and the well to be tested is shut in. The well is opened for production, ideally at constant flow rate, and the resulting pressure response is measured as the pressure draws down.

In a buildup test, the well has been producing (again, ideally at constant rate) for a period of time, thus creating a pressure gradient in the reservoir. The well is shut in, and the resulting pressure response is measured as the pressure builds up.

Exploration-Well Tests vs. Development-Well Tests. For a well test on an exploration well, the focus is on the entire reservoir. Objectives of an exploration-well test usually include fluid sampling, estimating initial reservoir pressure, evaluating well productivity, estimating distances to boundaries, and estimating fluids in place. Exploration-well tests often must be conducted while the rig is on location, making such tests extremely expensive. The expense is justified by the major investment decisions that will be made based on the information obtained from the test.

In contrast, in development-well testing, the focus is on the individual well and the near-wellbore area. Objectives of a well test on a development well might include estimating average drainage area pressure, evaluating stimulation treatment effectiveness, quantifying wellbore damage, and estimating reservoir permeability. Compared to exploration-well tests, development-well tests are relatively inexpensive. The results of development-well tests are used in minor investment decisions such as whether or not to do a stimulation treatment.

1.2.2 Applications and Objectives of Well Test Interpretation. Well test interpretation plays a role in many stages in the life of a well including exploration, appraisal, development, reservoir characterization, and production engineering.

Exploration and Appraisal. Well testing plays an extremely important role during exploration and appraisal. The information gained through well testing will be instrumental in making major investment decisions such as whether to set a platform, build a pipeline, develop a field, or sign a long-term production contract.

The two most important questions that well testing in exploration and appraisal wells can address are “How much oil or gas does this reservoir contain?” and “At what rate can wells in this reservoir produce?”

Flow tests are often used during exploration testing; these tests are designed to investigate enough of the reservoir to assess the economic viability of the prospect. The deliverability is obtained from the flow rates achieved during the test. Distances to boundaries, reservoir pore volume, and estimates of oil or gas in place may be obtained from the pressure-transient response.

Reservoir Engineering. In reservoir engineering, well testing may address questions such as “What is the in-situ permeability?,” “What are the nature of and distances to reservoir boundaries?,” and “What is the average reservoir pressure?”

A knowledge of the in-situ permeability is important in reservoir simulation and production forecasting. The nature of and distance to reservoir boundaries is important in reservoir simulation and in well-spacing decisions. The average reservoir pressure is used in reservoir simulation, well optimization, and material-balance calculations.

Production Engineering. The focus in production engineering is on the individual well. The questions addressed by well testing in production engineering reflect this focus: “Is the well damaged?” and “How effective was the stimulation treatment?”

Poor well performance may be caused by reservoir characteristics, such as low permeability or low reservoir pressure; by near-wellbore conditions, such as damage from mud-filtrate invasion, non-Darcy flow, or plugged perforations; or by some combination of these factors. Well test interpretation can help identify the cause of poor well performance, so that the appropriate remedial action may be taken.

1.3 Alternatives to Conventional Well Testing

There are several alternatives to pressure-transient testing that may also be used to estimate permeability and other reservoir properties. Alternatives include wireline formation testing, data from permanent gauges, rate-transient analysis, log-derived permeability estimates, and core analysis. In general, these methods supplement or complement, rather than replace, conventional pressure-transient testing.

1.3.1 Wireline-Formation Testing. In wireline formation testing, a relatively small amount of fluid (a few liters) is withdrawn from a short interval (1 m or less). The pressure response during both the withdrawal period and the subsequent shut-in period is recorded. Formation tests may be conducted open hole, with a small probe forced through the mudcake or with a short interval (typically 1 m) isolated by dual packers. Formation tests may even be conducted in cased hole, with the tool drilling through the casing and cement to provide hydraulic communication with the formation, collecting a fluid sample and pressure data, and plugging the hole in the casing in a single trip (Burgess et al. 2002).

Unlike conventional pressure-transient testing, formation testing can provide multiple small-scale measurements of in-situ permeability and pressure across the reservoir interval. Multiple downhole fluid samples may also be collected at different depths.

Because of the small volume of fluid withdrawn from the formation, wireline formation testing has a rather limited depth of investigation and provides little or no information about lateral reservoir boundaries. In high-permeability formations, the limited production volume may not create a large enough pressure response to allow formation permeability to be estimated. Normally, the small-scale permeability estimates from formation testing must be scaled up before use in forecasting well performance. Finally, skin factors obtained from wireline formation testing are of limited use for predicting well performance as completed for production (Elshahawi et al. 2008).

Applications of wireline formation testing include collection of downhole fluid samples, estimation of formation pressure and permeability as a function of depth, identification of depleted zones, and estimation of vertical permeability. Graphing pressure vs. depth allows the reservoir pressure gradient to be determined, thus enabling identification of the formation fluid and the location of fluid contact. Vertical interference testing provides estimates of vertical permeability, which is not usually available from conventional pressure-transient testing. Permeability estimates from wireline formation testing may also be used to calibrate correlations for estimating permeability from wireline logs.

Although wireline formation testing is considered an alternative to conventional testing, it relies on the same theoretical foundations as conventional pressure-transient testing. As a result, many of the methods discussed in this test find also direct application in wireline formation testing.

1.3.2 Permanent Monitoring. Permanently installed pressure gauges have become common in recent years. Permanent gauges offer a number of advantages over conventional testing but also have some disadvantages as well.

Perhaps the two primary advantages of permanent gauges are (1) they allow shut-in periods that occur as a result of normal production operations to be analyzed as buildup tests, thereby eliminating the expense of loss or deferral of production, and (2) they provide frequent tests allowing continual monitoring of skin factor, reservoir permeability, and movement of fluid contacts, allowing timely remedial action.

On the other hand, data quality from permanent gauges obtained during routine production operations is not as high as that from conventional tests run under controlled conditions. Permanent monitoring must include both rate and pressure measurements to be useful. Because of the large quantity of data that is acquired, automated preprocessing and analysis are essential to realize their full potential.

Like wireline formation testing, interpretation of permanent gauge data is based on the same theoretical foundation as conventional pressure-transient testing.

1.3.3 Rate-Transient Analysis. Rate-transient analysis uses production and pressure data obtained during normal production operations to estimate reservoir properties. Methods used for rate-transient analysis are very similar to those used for pressure-transient analysis. Under the right conditions, rate-transient analysis can provide estimates of initial hydrocarbons in place, reservoir permeability, and skin factor (for vertical wells) or fracture half length (for hydraulically fractured wells).

Advantages of rate-transient analysis over pressure-transient analysis include lower cost of data collection (no special equipment, no need to shut in the well, no need to interrupt normal operations), and the ability to reach a much larger radius of investigation than could be obtained by pressure-transient testing. This latter advantage has made rate-transient analysis one of the most important reservoir characterization tools for low permeability formations.

While pressure-transient data is collected under controlled conditions, specifically for the purpose of estimating reservoir properties, data used for rate-transient analysis is collected during normal production operations. Thus, rate-transient data are typically noisier and are more likely to be subject to changes in operating conditions than are pressure-transient data. In addition, early-time data are often inadequate to reliably distinguish between damaged wells and undamaged wells. With rate-transient analysis, it is also difficult to track changes in skin factor, effective permeability, and drainage pressure over time.

The primary application of rate-transient analysis is estimation of permeability, stimulation effectiveness, and drainage area or original hydrocarbons in place, for moderate to low permeability wells that are produced at capacity. Rate-transient analysis is particularly important for estimating original hydrocarbons in place and reserves in low permeability reservoirs where it is difficult (if not impossible) to estimate average reservoir pressures for material balance analysis.

Rate-transient analysis is based on the same foundations as conventional well testing, wireline formation testing, and analysis of permanent gauge data.

1.3.4 Log-Derived Permeability Estimates. In many reservoirs, it may be possible to develop correlations to estimate permeability from a standard logging suite. Although not applicable to all reservoirs, log-derived permeability estimates may be quite useful. Typically, core data will be used to identify two or more distinct rock types. Separate correlations are then developed for each rock type. The correlations are calibrated with permeability estimates from wireline-formation testing or core analysis.

Unlike fluid property correlations, log permeability correlations have to be custom built for each rock type in each formation. In some cases, it may not be possible to develop correlations that are accurate enough to be useful. In other cases, the correlations may serve as permeability indicators rather than reliable permeability estimates. In older fields, core or formation test data to calibrate correlations may not be available. As with permeability estimates from wireline formation testing or from core analysis, log-derived permeabilities must be scaled up to use in preformation forecasting or comparison with permeability estimates from conventional pressure-transient tests.

If feasible, log-derived permeability estimates allow a limited amount of formation test or core data to be extrapolated to all wells that have a standard suite of logs.

1.3.5 Core Analysis. Conventional core analysis provides measurements of fluid saturations, porosity, and absolute permeability. Depending on the specific tests requested, special core analysis may provide measurements of capillary pressure, relative permeability, and stress-dependent porosity and/or permeability.

Although core analysis is an essential component of a complete reservoir study, it is often difficult to relate laboratory-derived permeabilities to those obtained from pressure-transient test interpretation or required for performance forecasting. Some of the reasons for this difficulty include

- Differences in scale—Core plug sizes are typically measured in centimeters or inches; pressure-transient testing measures permeabilities over distances of tens of feet to several hundred feet.
- Differences in conditions—Conventional core analysis permeability measurements are conducted at room temperature and low confining pressures. Pressure-transient testing provides estimates of permeability at in-situ temperature and stress conditions.
- Differences in fluids—Conventional core analysis permeability measurements are typically conducted using air under single-phase conditions. Pressure-transient testing provides estimates of permeability to reservoir fluids with reservoir fluid saturations. The combined effects of stress and connate water may cause the in-situ permeability for low permeability or stress-sensitive formations may be two or three orders of magnitude lower than that measured on unstressed, dry cores (Jones and Owens 1980).
- Incomplete core recovery—Incomplete core recovery may be particularly significant in unconsolidated sands or in thick formations.
- Fracture permeability not captured—Virtually all of the bulk permeability in naturally fractured reservoirs is provided by the fracture network. If the fractures are widely spaced, a core may not intersect a representative sample of the fractures. Even when a whole core cuts a natural fracture, it may not be possible to determine the in-situ fracture aperture and thus, the permeability.

- Improper lab technique—The use of inappropriate lab technique may cause irreversible changes in the core during cleaning and drying.
- Biased core plug sampling—If lab personnel are inexperienced or lack adequate training, cores plugs may be preferentially selected from the more permeable portions of the whole core.

1.4 Forward and Inverse Problems

In the *forward* (or direct) problem, **Fig. 1.1**, the input flow rate history and the system reservoir model are both known, and the resulting pressure response must be determined. Any given reservoir model will always give the same output pressure response for a given input flow rate history. In other words, the solution to the forward problem is unique.

In contrast, in the *inverse* problem, **Fig. 1.2**, the input flow rate history and output pressure response are known, while the unknown reservoir model must be found. Many different reservoir models may give essentially the same pressure response for a given input flow rate history. The solution to the inverse problem is *not* unique.

A forward (or direct) problem has a unique solution. For any given input and model, the output will always be the same.

An inverse problem does not have a unique solution; many different models can produce the same output for a given input.

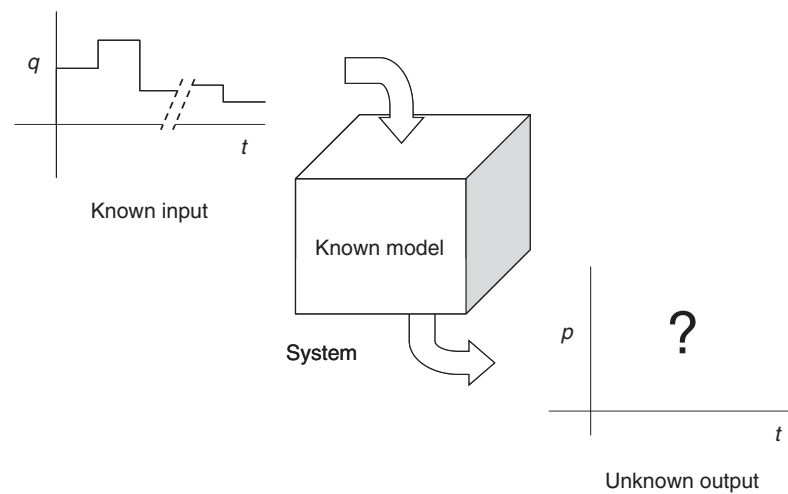


Fig. 1.1—Forward problem.

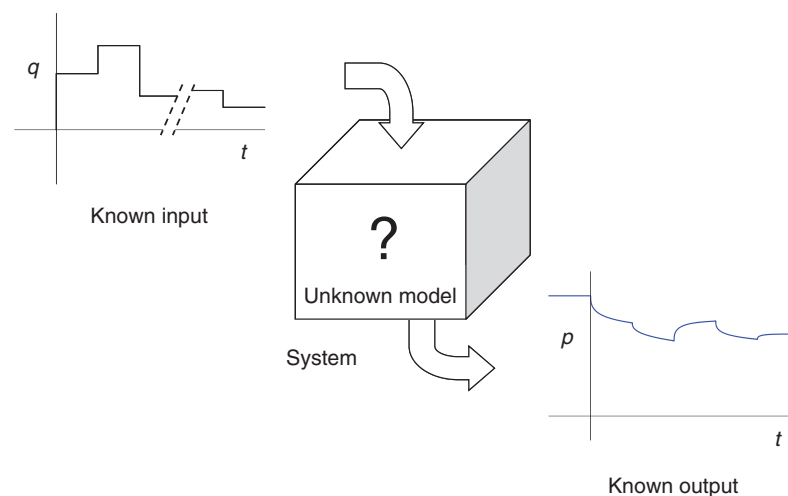


Fig. 1.2—Inverse problem.